HEAT EXCHANGE IN A TUBE IN THE PRESENCE OF VIBRATIONAL RELAXATION IN THE GAS

M. S. Povarnitsyn

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The temperature distribution of a heat-liberating wall of a circular tube, in which a Poisseuille gas flows, is obtained. The influence of nonequilibrium excitation and relaxation of the vibrational degrees of freedom of the diatomic gas on the wall temperature is taken into account.

When investigating high-temperature heat exchangers it is of interest to examine the influence on the temperature distribution in the gas stream and on the walls of thermal excitation and relaxation of the vibrational degrees of freedom of diatomic gas molecules in frozen dissociation.

There are several papers in the literature, for example, [1-6], studying the influence on the gas temperature of excitation of the vibrational degrees of freedom of molecules in gas streams flowing over a plate or an axisymmetric body near the stagnation point, and also studies of the relaxation in shockwaves.

However, the heat exchange for the internal flow of a relaxing gas has been investigated less. The hydrodynamically developed laminar flow in a tube is considered here, which in the case $\theta_0 = (T_S - T_0)/T_0 \leq 0.25$ and $T_0 \leq T \leq T_S$ can be described by the equations of vibrational energy transfer, heat conduction, and dissociated atom diffusion [4]:

$$u\frac{\partial \varepsilon}{\partial x} = D_{22}\left(\frac{\partial^2 \varepsilon}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial \varepsilon}{\partial r}\right) + \frac{\tilde{\varepsilon} - \varepsilon}{\tau_V} + w_{k-d}, \tag{1}$$

$$\rho u c_p \frac{\partial T}{\partial x} = \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r}\right) + u\frac{\partial P}{\partial x} + \eta\left(\frac{\partial u}{\partial r}\right)^2 - \frac{\tilde{\varepsilon} - \varepsilon}{\tau_V} - w_{k-d} - \rho\frac{E_D}{\mu} w_A, \tag{2}$$

$$\rho u \frac{\partial c}{\partial x} = \rho D_{12}\left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial c}{\partial r}\right) + \rho w_A, \tag{3}$$

where $w_{k-d} = \bar{G} w_r m_2^{-2} - E w_d m_2^{-1}$; c is the concentration of atoms, ρ is the density, m_2 is the mass of the molecule. The rate of production of the dissociated atoms is given by

$$w_A = 4k_r\left(\frac{P}{RT}\right)^2\left[\frac{c_{eq}^2}{1 - c_{eq}^2}(1 - c) - \frac{c^2}{1 + c}\right] \equiv (w_d - w_r)\rho^{-1}.$$

The expression for the rate of production of the vibrational energy $(\tilde{\varepsilon} - \varepsilon)\tau_V^{-1}$ is of a general character, independent of the kind of molecules, the gas composition, and the excitation of the diverse oscillator levels [1]. Further notation in (1)-(3): ε is the vibrational energy of unit volume; $\tilde{\varepsilon}$ is the equilibrium vibrational energy, $\tilde{\varepsilon} = Nh'\omega(\exp\theta/T - 1)^{-1} = \rho(R/\mu)\theta(\exp\theta/T - 1)^{-1}$. The characteristic temperature $\theta = h'\omega/k$, $h'\omega$ is the energy of a vibrational quantum transition from one level to the next. We have [6]: θ = 2240°K for O_2, 3354°K for N_2, 5910°K for H_2, 810°K for Cl_2, 4830°K for D_2. Here h', k are the Planck and Boltzmann constants.

It is convenient to use the following semiempirical formula [6] for the relaxation time

$$\lg\frac{P\tau}{\tau'_\theta} = a_T\left[\left(\frac{\theta}{T}\right)^{1/3} - 1\right]. \tag{4}$$

For example, for a number of gases τ'_θ is equal to (μsec·atm): Cl_2–0.265, O_2–3.95, N_2–18, D_2–0.128 [6], and for these gases $a_T = 3.211$. For hydrogen [8] $\tau'_\theta = 1.0\cdot10^{-7}$ sec·atm, $a_T = 2.40$.

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Estimates show that the term w_{k-d} for the interaction between dissociation and vibrational relaxation can be neglected for weak dissociation ($c \leq 0.1$).

Let us consider a flow frozen with respect to dissociation in a tube with noncatalytic walls $w_{k-d} = w_A = 0$.

Hence, using dimensionless variables we deduce from (1)-(3)

$$\mathrm{Le}^{-1}(1-\xi^2)\frac{\partial e}{\partial \tau} = \frac{\partial^2 e}{\partial \xi^2} + \frac{1}{\xi}\cdot\frac{\partial e}{\partial \xi} + \psi_h(\tilde{e}-e), \tag{5}$$

$$(1-\xi^2)\frac{\partial \vartheta}{\partial \tau} - \varepsilon_0(1-2\xi^2) = \frac{\partial^2 \vartheta}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial \vartheta}{\partial \xi} + \mathrm{Le}\,\psi_h(\tilde{e}-e), \tag{6}$$

where

$$\psi_h = h^2 P^2 (D_{00}\tau_\theta')^{-1} \exp\left[-\alpha_\tau\left(\frac{\theta}{T}\right)^{1/3} + \alpha_\tau\right]\left(\frac{T_0}{T}\right)^{1.81}, \tag{7}$$

D_{022} is the coefficient of molecular self-diffusion for $T = T_0$ and $P = 1$ atm, $\alpha_T = 2.302 a_T$

In the temperature range under consideration $T_0 < T < T_S$ $(0 < \vartheta < 1)$, as computations show, an approximation

$$\psi_h(\tilde{e}-e) \simeq \psi_0(a + b_1\vartheta - e) \tag{8}$$

can be introduced to the accuracy of several percent. Hence $\psi_0 \sim P^2$, and the coefficients a, b_1 are independent of the pressure. Substituting (8) into (5) and (6), and considering the case Le = 1, let us deduce the initial system

$$(1-\xi^2)\frac{\partial e}{\partial \tau} = \frac{1}{\xi}\cdot\frac{\partial e}{\partial \xi} + \frac{\partial^2 e}{\partial \xi^2} + \psi_0(a + b_1\vartheta - e), \tag{9}$$

$$(1-\xi^2)\frac{\partial \vartheta}{\partial \tau} - \varepsilon_0(1-2\xi^2) = \frac{1}{\xi}\cdot\frac{\partial \vartheta}{\partial \xi} + \frac{\partial^2 \vartheta}{\partial \xi^2} + \psi_0(a + b_1\vartheta - e) \tag{10}$$

with conditions on the leading edge of the tube

$$\vartheta(0,\ \xi) = 1, \quad e(0,\ \xi) = \tilde{e}(T_0) \equiv e_0 \tag{11}$$

and on the wall for $\xi = 1$

$$\frac{\partial \vartheta}{\partial \xi} = -g, \quad \frac{\partial e}{\partial \xi} = 0. \tag{12}$$

The problem (9)-(12) differs from the considered problem of heat exchange in a dissociating gas flowing in a plane channel [9] only by its symmetry.

Let us form the function $f_1 = \vartheta - e$, for which we obtain an equation from (9)-(10):

$$(1-\xi^2)\frac{\partial f_1}{\partial \tau} - \varepsilon_0(1-2\xi^2) = \frac{\partial^2 f_1}{\partial \xi^2} + \frac{1}{\xi}\cdot\frac{\partial f_1}{\partial \xi} \tag{13}$$

with boundary conditions $f_1 = 1 - e_0$ for $\tau = 0$, and $\partial f_1/\partial\xi = -g$ for $\xi = 1$. The solution of the problem (13) is equivalent to the solution of the heat conduction equation (10) with the condition $\psi_0 = 0$. To solve this temperature problem let us go over to a Laplace transform in τ

$$(1-\xi^2)(p\vartheta^* - 1) = \frac{1}{\xi}\cdot\frac{d\vartheta^*}{d\xi} + \frac{d^2\vartheta^*}{d\xi^2} + \frac{\varepsilon_0(1-2\xi^2)}{p}. \tag{14}$$

Denoting the solution of the homogeneous equation by $\varphi(\xi, \alpha)$, we write the complete solution of (14) thus [10]:

$$\vartheta^* = \left[d + \int_0^{\xi} \xi_1^{-1}\varphi^{-2}d\xi_1 \int_0^{\xi_1} \xi_2\varphi h\,d\xi_2\right]\varphi(\xi,\ \alpha), \tag{15}$$

where

$$h = \xi^2 - 1 + \frac{\varepsilon_0}{\alpha}(1-2\xi^2). \tag{16}$$

The homogeneous equation is similar to the Whittaker equation [10] and has the solution

$$\varphi = \exp\left(-\frac{\sqrt{\alpha}}{2}\xi^2\right){}_1F_1\left(b_0,\ 1,\ \sqrt{\alpha}\,\xi^2\right),\quad b_0 = \frac{2-\sqrt{\alpha}}{4}. \tag{17}$$

Expanding φ in a series shows that this is an entire function of the arguments ξ^2 and α:

$$\varphi = 1 - \frac{\alpha}{4}\xi^2 + \left(\frac{\alpha}{16} + \frac{\alpha^2}{64}\right)\xi^4 - \frac{5\alpha^2}{9\cdot 64}\xi^2 + \frac{\alpha^2\xi^8}{16\cdot 64} + O(\alpha^3). \tag{18}$$

The coefficient d is determined from the condition on the wall:

$$d = -\frac{1}{\varphi'(1)}\left[gp^{-1} + \varphi'(1)\int_0^1 \xi_1^{-1}\varphi^{-2}d\xi_1\int_0^{\xi_1}\xi_2\varphi h d\xi_2 + \varphi^{-1}(1)\int_0^1 \xi\varphi h d\xi\right]. \tag{19}$$

Here $\varphi(1) \equiv \varphi(1,\ \alpha)$; $\varphi'(1) \equiv \partial\varphi/\partial\xi$ for $\xi = 1$.

The value of the wall temperature in the transformed plane is

$$\vartheta^*(1) = -\frac{g\varphi(1)}{p\varphi'(1)} - \frac{\int_0^1 \xi\varphi h d\xi}{\varphi'(1)}.$$

Since $\vartheta^* = -\alpha^{-1}$ for $g = \varepsilon_0 = 0$, then

$$\vartheta^*(1) = -\frac{1}{\alpha} + \frac{g\varphi(1)}{\alpha\varphi'(1)} - \frac{\int_0^1 \xi\varphi h_0 d\xi}{\varphi'(1)},\quad h_0 = \frac{\varepsilon_0}{\alpha}(1-2\xi^2). \tag{20}$$

We find [11]

$$\varphi'(1) = \sqrt{\alpha}\exp\left(-\frac{\sqrt{\alpha}}{2}\right)\left[{}_1F_1\left(b_0,\ 1,\ \sqrt{\alpha}\right) - \frac{2+\sqrt{\alpha}}{2}\,{}_1F_1\left(b_0,\ 2,\ \sqrt{\alpha}\right)\right],$$

$$\frac{\partial\,{}_1F_1\left(b_0,\ \gamma,\ \sqrt{\alpha}\right)}{\partial\alpha} = \frac{1}{2\sqrt{\alpha}}\sum_{k=1}^{\infty}\frac{b_0(b_0+1)\ \ldots\ (b_0+k-1)\,\alpha^{k/2}}{\gamma(\gamma+1)\ \ldots\ (\gamma+k-1)\,k!}\varphi_k \equiv \frac{1}{2\sqrt{\alpha}}\Phi\left(b_0,\ \gamma,\ \sqrt{\alpha}\right), \tag{21}$$

$$\varphi_k = \frac{k}{\sqrt{\alpha}} - \frac{1}{4}\sum_{l=1}^{k}(b_0 + l - 1)^{-1}.$$

Hence, we deduce the temperature on the tube wall

$$\vartheta(\tau,\ 1) = \vartheta^0(\tau) - g\sum_{m=1}^{\infty}\left(a_m + \frac{\varepsilon_0}{g}b_m\right)\exp(-\alpha_m\tau), \tag{22}$$

where

$$a_m = {}_1F_1\left(b_0,\ 1,\ \sqrt{\alpha_m}\right)/\psi_m(\alpha);$$

$$b_m = -\exp\left(\sqrt{\alpha_m}/2\right)\int_0^1 \xi(1-2\xi^2)\,\varphi d\xi/\psi_m(\alpha).$$

The function $\psi_m(\alpha)$ is given by (46) later. The sum is taken over the roots $\alpha(\alpha > 0)$ of the equation

$${}_1F_1\left(b_0,\ 1,\ \sqrt{\alpha}\right) - \frac{2+\sqrt{\alpha}}{2}\,{}_1F_1\left(b_0,\ 2,\ \sqrt{\alpha}\right) = 0. \tag{23}$$

In particular, we have $\alpha_1 = 25.674$; $\alpha_2 = 83.847$; $\alpha_3 = 174.16$; $a_1 = -0.1986$; $a_2 = -0.06967$; $a_3 = -0.03652$; $b_1 = -0.04032$; $b_2 = -0.00881$; $b_3 = -0.00174$.

Using (18) and the expansion of the function $\varphi'(1)$, let us calculate the residues of (20) multiplied by $\exp p\tau$ near the pole $p = 0$ of multiplicity 1 and 2, which yields

$$\vartheta^0(\tau) = 1 - 4\tau g - \frac{11}{24}g - \frac{1}{16}\varepsilon_0. \tag{24}$$

Furthermore, let us calculate the temperature of the tube walls taking account of excitation of the vibrational gas energy at $\varepsilon_0 = 0$. The solution of (13) for the function $f = f_1(1-e_0)^{-1}$ reduces to solving the temperature problem (15), (19) using the substitution $g' = g(1-e_0)^{-1}$. We deduce

$$f^*(\xi,\ \alpha) = -\frac{1}{\alpha} + \frac{g'\varphi(\xi,\ \alpha)}{\alpha\,(\partial\varphi/\partial\xi)_{\xi=1}}\,. \tag{25}$$

After eliminating the vibrational energy by using f_1, the equation for the temperature (9) is

$$(1-\xi^2)\,\frac{\partial\vartheta}{\partial\tau} = \frac{1}{\xi}\cdot\frac{\partial\vartheta}{\partial\xi} + \frac{\partial^2\vartheta}{\partial\xi^2} + \psi_0\,(a + b\vartheta + f_1(\tau,\ \xi)) \tag{26}$$

under the conditions (11), (12). Here $b = b_1 - 1$. The Laplace transform of (26) in τ yields the equation

$$\frac{d^2\vartheta^*}{d\xi^2} + \frac{1}{\xi}\cdot\frac{d\vartheta^*}{d\xi} = \left[p\,(1-\xi^2) - \psi_0 b\right]\vartheta^* - \left[(1-\xi^2) + \frac{a\psi_0}{p}\right] - \psi_0 f_1^*\,. \tag{27}$$

Let us write the solution of the homogeneous equation [10]

$$\varphi_\psi = \exp\left(-\frac{\sqrt{\alpha}}{2}\,\xi^2\right)\,{}_1F_1\,(b_\psi,\ 1,\ \sqrt{\alpha}\,\xi^2),\quad b_\psi = \frac{1}{2} - \frac{b\psi_0 + \alpha}{4\sqrt{\alpha}}\,. \tag{28}$$

The solution of the inhomogeneous equation (27) is given by (15) with φ replaced φ_ψ and h by

$$h_\psi = \xi^2 - 1 + \frac{a\psi_0}{\alpha} - \psi_0 f_1^*\,. \tag{29}$$

Since the walls are noncatalytic (12), then the expression for the coefficient d is given by (19) with the substitutions mentioned.

The gas at the entrance is in thermodynamic equilibrium. Hence, $\beta + \gamma + \psi_0(1-e_0) = 0$, where $\beta = b\psi_0$, $\tilde{\gamma} = a\psi_0$. Using the mentioned equality and the condition $\vartheta^* = -\alpha^{-1}$ for $g = 0$, we find the wall temperature in the transformed plane

$$\vartheta^*(1) = -\frac{1}{\alpha} + g\alpha^{-3/2}\left[{}_1F_1\,(b_\psi,\ 1,\ \sqrt{\alpha}) + \frac{\beta\exp\,(\sqrt{\alpha}/2)}{b\,(\partial\varphi/\partial\xi)_{\xi=1}}\int\limits_0^1 \xi\varphi_\psi\varphi\,d\xi\right]$$
$$\times\left[{}_1F_1\,(b_\psi,\ 1,\ \sqrt{\alpha}) - \left(1 + \frac{\alpha+\beta}{2\sqrt{\alpha}}\right){}_1F_1\,(b_\psi,\ 2,\ \sqrt{\alpha})\right]^{-1}. \tag{30}$$

First let us calculate the developed temperature distribution. Let us find the residues at the pole $\alpha = 0$, for which we obtain the expansions

$$\begin{aligned} {}_1F_1\,(b_\psi,\ 1,\ \sqrt{\alpha}) &= h_0 + h_1\sqrt{\alpha} + h_2\alpha + \ldots, \\ {}_1F_1\,(b_\psi,\ 2,\ \sqrt{\alpha}) &= f_0 + f_1\sqrt{\alpha} + f_2\alpha + \ldots, \end{aligned} \tag{31}$$

where in particular

$$\begin{gathered} h_0(\beta) = J_0\,(\sqrt{\beta}),\quad h_1 = h_0/2, \\ f_0 = 2\beta^{-1/2}J_1\,(\sqrt{\beta}),\quad f_1 = \beta^{-1/2}\,[J_1(\sqrt{\beta}) - 2\beta^{-1/2}J_2\,(\sqrt{\beta})]\,. \end{gathered} \tag{32}$$

Let Π_1 denote the second and Π_2 the third member in (30) and let us establish their behavior near the pole $\alpha = 0$. We obtain the expansion needed

$$\prod_{\substack{1\\(\alpha\to 0)}} = -gh_0\,(\alpha f_0)^{-1}\left[\frac{\beta}{2} + \left(\frac{1}{2} - a_1 - \frac{\beta a_2}{2}\right)\alpha + O\,(\alpha^2)\right]^{-1}, \tag{33}$$

where $a_1 = 0.5 - f_1 f_0^{-1}$, $a_2 = h_2 h_0^{-1} - h_1^2 h_0^{-2} + f_1(2f_0)^{-1} - f_2 f_0^{-1}$.

To obtain $\prod\limits_{\alpha\to 0}{}_2\,(\alpha,\ \beta)$, let us expand the functions φ, φ_ψ in a series in α:

$$\varphi_\psi\,(\alpha,\ \xi) = h_0\,(\xi,\ \beta) + \left[h_2\,(\beta,\ \xi) - \frac{\xi^4}{8}\,h_0\,(\xi,\ \beta)\right]\alpha + O\,(\alpha^2), \tag{34}$$

$$\varphi\,(\alpha,\ \xi)\,\varphi_\psi\,(\alpha,\ \xi) = h_0\,(\xi,\ \beta) + \left[h_2\,(\xi,\ \beta) - \frac{h_0\,(\xi,\ \beta)}{4}\left(\frac{\xi^4}{4} + \xi^2\right)\right]\alpha + O\,(\alpha^2).$$

Let us form

$$\int_0^1 \varphi_\psi \varphi \xi d\xi = \frac{1}{\sqrt{\beta}} J_1(\sqrt{\beta}) + \alpha \Xi + O(\alpha^2). \tag{35}$$

Remarking that $\int_0^1 h_2 d\xi = 0.5 f_2(\alpha)$, we obtain from (34)

$$\Xi = \frac{f^2}{2} + \frac{1}{8} \sum_{n=0,1,\ldots}^{\infty} \frac{(-1)^{n+1}\beta^n}{4^n (n!)^2} \left(\frac{1}{4(3+n)} + \frac{1}{2+n} \right). \tag{36}$$

We hence find the needed expansion

$$\prod_{(\alpha\to 0)}^{2} = \frac{8g}{bf_0\alpha^2} \cdot \frac{\beta^{-1/2} J_1(\sqrt{\beta}) + \alpha\Xi + O(\alpha^2)}{1 + \alpha\chi_0 + O(\alpha^2)}, \tag{37}$$

$$\chi_0 = -\frac{19}{96} + \frac{1}{2}\left(\frac{1}{2} - \frac{f_1}{f_0}\right) + \frac{\beta f_2 + 2f_1}{\beta f_0}. \tag{38}$$

Multiplying Π_1 and Π_2 from (33) and (37) by $\exp p\tau$ and evaluating the residues at the pole $\alpha = 0$, we derive an expression for the asymptotic wall temperature ($\tau \gg 1$)

$$\frac{1 - \vartheta^0(\tau)}{g} = \frac{4}{b}\left(\Xi \frac{\sqrt{\beta}}{J_1(\sqrt{\beta})} - \chi_0\right) - \frac{J_0(\sqrt{\beta})}{\sqrt{\beta} J_1(\sqrt{\beta})} - \frac{4\tau}{b}. \tag{39}$$

Substituting (36) and (38) into (39), we notice that the function $f_2(\beta)$ is eliminated. We derive

$$[1 - \vartheta^0(\tau)]\, g^{-1} = c_\tau - 4\tau b^{-1}, \tag{40}$$

$$c_\tau = \frac{1}{b}\left[\frac{\sqrt{\beta}}{2J_1(\sqrt{\beta})} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}\beta^n}{4^n(n!)^2} \left(\frac{1}{4(3+n)} + \frac{1}{2+n} \right) + \frac{19}{24} - \frac{8}{\beta} \right.$$
$$\left. + \frac{16}{\beta^2} + \frac{J_0(\sqrt{\beta})}{\sqrt{\beta} J_1(\sqrt{\beta})} \left(2 - \frac{8}{\beta}\right) \right] - \frac{J_0(\sqrt{\beta})}{\sqrt{\beta} J_1(\sqrt{\beta})}. \tag{41}$$

The undeveloped wall temperature distribution is given by the formula $\vartheta(\tau) = \vartheta^0 + \vartheta^1(\tau)$, where $\vartheta^1(\tau)$ is determined by the sums of residues of the expressions $(\Pi_1 + \Pi_2) \times \exp p\tau$ over the roots α, $\alpha^0(\alpha, \alpha^0 > 0)$ of the equations

$${}_1F_1(b_\psi, 1, \sqrt{\alpha}) - \left(1 + \frac{\alpha + \beta}{2\sqrt{\alpha}}\right) {}_1F_1(b_\psi, 2, \sqrt{\alpha}) = 0, \tag{42}$$

$${}_1F_1\left(\frac{2\sqrt{\alpha^0}}{4}, 1, \sqrt{\alpha^0}\right) - \frac{2+\sqrt{\alpha^0}}{2} {}_1F_1\left(\frac{2-\sqrt{\alpha^0}}{2}, 2, \sqrt{\alpha^0}\right) = 0, \tag{43}$$

obtained by equating the denominators of Π_1 and Π_2 in (30) to zero. The solution for the wall is written as

$$\frac{1 - \vartheta(\tau)}{g} = c_\tau - \frac{4\tau}{b} + \sum_{m=1}^{\infty} \left[a_m \exp(-\alpha_m \tau) + a_m^0 \exp(-\alpha_m^0 \tau) \right], \tag{44}$$

where

$$a_m = \frac{{}_1F_1(b_\psi, 1, \sqrt{\alpha})}{\Phi_m} + \frac{\beta}{b\sqrt{\alpha}} \frac{(\exp\sqrt{\alpha}) \int_0^1 \varphi_\psi \varphi \xi d\xi}{\Phi_m \left[{}_1F_1(b_0, 1, \sqrt{\alpha}) - \frac{2+\sqrt{\alpha}}{2} {}_1F_1(b_0, 2, \sqrt{\alpha}) \right]};$$

$$a_m^0 = \frac{\beta}{b} \left\{ \int_0^1 \varphi_\psi \varphi \xi d\xi \exp\sqrt{\alpha^0} \right\} \left\{ \sqrt{\alpha^0}\, \psi_m(\alpha^0) \left[{}_1F_1(b_\psi, 1, \sqrt{\alpha^0}) - \left(1 + \frac{\alpha^0 + \beta}{2\sqrt{\alpha^0}}\right) {}_1F_1(b_\psi, 2, \sqrt{\alpha^0}) \right] \right\}^{-1}; \tag{45}$$

$$\Phi_m = \frac{\alpha}{2} \left[\Phi(b_\psi, 1, \sqrt{\alpha}) - \frac{\alpha - \beta}{2\alpha} {}_1F_1(b_\psi, 2, \sqrt{\alpha}) - \left(1 + \frac{\alpha + \beta}{2\sqrt{\alpha}}\right) \Phi(b_\psi, 2, \sqrt{\alpha}) \right]; \tag{46}$$

$$\psi_m = \frac{\alpha}{2} \left[\Phi(b_0, 1\ \sqrt{\alpha}) - \frac{1}{2} {}_1F_1(b_0, 2, \sqrt{\alpha}) - \frac{2+\sqrt{\alpha}}{2} \Phi(b_0, 2, \sqrt{\alpha}) \right];$$

$$\Phi(b, \gamma, \sqrt{\alpha}) = \sum_{k=1}^{\infty} \frac{b(b+1)\ldots(b+k-1)\,\alpha^{k/2}}{\gamma(\gamma+1)\ldots(\gamma+k-1)\,k!} \varphi_k(b);$$

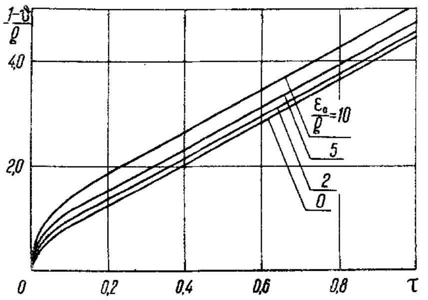

Fig. 1

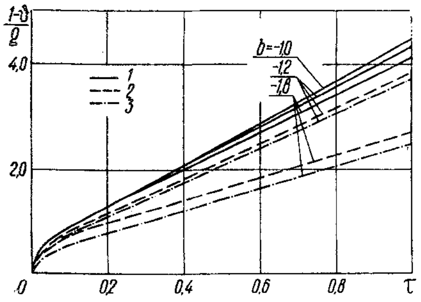

Fig. 2

Fig. 1. Dimensionless temperature of a tube wall without taking account of vibrational relaxation for different intensities of heat evolution due to internal friction and the work of pressure forces.

Fig. 2. Dimensionless temperature of a tube wall for different values of the reaction rate constant for excitation of molecule vibrations and the angular coefficient b: 1) $\beta = -0.2$; 2) -5; 3) -70.

$$\varphi_k(b) = \frac{k}{\sqrt{\alpha}} + 2\sqrt{\alpha}\,\frac{\partial b}{\partial \alpha}\sum_{l=1}^{k}(b+l-1)^{-1}.$$

For brevity the subscript m is omitted in (45) and (46).

The distribution of the dimensionless wall temperature $(1-\vartheta)g^{-1}$ obtained by (22) is given in Fig. 1 as a function of the coordinate τ and the ratio ε_0/g characterizing the heat to internal friction and the work of the pressure forces. Excitation of molecule vibrations is not taken into account here. The dependence $[1-\vartheta(\tau)]g^{-1}$ computed by means of (40)-(46) is shown in Fig. 2 for different values of the parameters β and b characterizing the nonequilibrium rate of the excitation and relaxation processes for molecule vibrations. These processes are excluded for $b = -1$. As the rate of excitation grows ($|\beta| \sim P^2$ grows), the wall temperature is reduced noticeably.

NOTATION

b_1	is the angular coefficient of the approximation (8);
$b = b_1 - 1$; $b_0 = (2-\sqrt{\alpha})/4$; $b_\psi = 1/2-(\beta+\alpha)/4\sqrt{\alpha}$;	
D_{22}, D_{12}	are coefficients of self and mutual diffusion;
$e = \varepsilon/\rho_0 c_p(T_S - T_0)$	is the dimensionless vibrational energy;
$\tilde{e} = (\gamma-1)\theta/T_\gamma\theta_0(\exp(\theta/T)-1)$;	
E_D	is the dissociation energy of a mole of gas;
$\bar{E}$	is the energy of the mean molecule level at which atoms recombine;
$f_1 = \vartheta - e$, $f = f_1(1-e_0)^{-1}$;	
${}_1F_1(b, \gamma, x)$	is a degenerate hypergeometric function;
G	is the energy of the mean molecule level from which dissociation occurs;
G	is the heat flux density at the wall;
$g = GR/\lambda(T_S - T_0)$;	
P	is the pressure;
p	is the variable in the Laplace transform;
R	is the tube radius
$0 \le r \le R$;	
T	is the temperature in the tube;
$T_0 \le T \le T_S$;	
T_0	is the temperature at the channel entrance;
T_S	is the previously selected temperature;
$(T_S - T_0)T_0^{-1} \equiv \theta_0 \le 0.25$;	
u	is the longitudinal velocity;
u_m	is the mean velocity;

x	is the coordinate along the tube;
$\alpha \equiv -p$; $\beta = b\psi_0$; $\gamma = c_p/c_v$;	
ε	is the vibrational energy of a unit volume;
$\varepsilon_0 = 16M^2 Pr(\gamma-1)\theta_0^{-1}$;	
λ	is the coefficient of heat conduction of the gas;
η	is the velocity;
μ	is the molecular weight;
$\vartheta = (T_s - T)/(T_s - T_0)$; $\vartheta^* = \quad \vartheta \exp(-p\tau)d\tau$;	
τ_v	is the vibrational relaxation time;
$\tau = x/2R\,Pr\,Re$; $\xi = r/R$; $Le = Pr = c_p\lambda^{-1}\eta$; $Sc = \eta/\rho D_{22}$;	
$Re = \rho u_m R\eta^{-1}$	is the Reynolds number taken for mean values of the temperature and pressure is the channel;
J_n	is the Bessel function.

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